

# Three-quark currents and baryon spin

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We show that three-quark axial currents as required by broken SU(6) spin-flavor symmetry reduce the quark spin contribution to proton spin from  $\Sigma_p = 1$  (one-quark axial current value) to  $\Sigma_p = 0.41(12)$  consistent with the empirical value  $\Sigma_{p,exp} = 0.33(08)$ . In the case of the  $\Delta^+(1232)$  baryon, we find that three-quark axial currents increase the one-quark axial current value  $\Sigma_{\Delta^+} = 3$  to  $\Sigma_{\Delta^+} = 3.87(22)$ . We also calculate the quark orbital angular momenta  $L_u$  and  $L_d$  in the proton and  $\Delta^+$  and interpret our results in terms of the prolate and oblate geometric shapes of these baryons consistent with their intrinsic quadrupole moments.

## I. INTRODUCTION

The question how the proton spin is made up from the quark spin  $\Sigma$ , quark orbital angular momentum  $L_q$ , gluon spin  $S_g$ , and gluon orbital angular momentum  $L_g$

$$J = \frac{1}{2}\Sigma + L_q + S_g + L_g \quad (1)$$

is one of the central issues in nucleon structure physics [1, 2]. In the constituent quark model with only one-quark operators, also called additive quark model, one obtains  $J = \Sigma/2 = 1/2$ , i.e., the proton spin is the sum of the constituent quark spins and nothing else. However, experimentally it is known that only about 1/3 of the proton spin comes from quarks [3]. The disagreement between the additive quark model result and experiment came as a surprise because the same model accurately described the related proton and neutron magnetic moments. We show that the failure of the additive quark model to describe the quark contribution to proton spin correctly is due to its neglect of three-quark terms in the axial current [4].

## II. BROKEN SPIN-FLAVOR SYMMETRY AND QCD PARAMETRIZATION METHOD

In the present work, the general QCD parametrization method developed and explained in detail by Morpurgo [5] is used to calculate the quark contribution to baryon spin in a systematic manner. Previously, we have applied this method to calculate higher order corrections to baryon-meson couplings [6] and baryon electromagnetic moments [7]. Here, we construct the most general expression for the quark angular momentum operator  $\tilde{\Omega}$

in spin-flavor space that is compatible with the space-time and inner QCD symmetries.

The first step is to realize [8] that a general SU(6) spin-flavor operator  $\tilde{\Omega}^R$  acting on the **56** dimensional baryon ground state supermultiplet must transform according to one of the irreducible representations  $R$  contained in the direct product  $\mathbf{56} \times \mathbf{56} = \mathbf{1} + \mathbf{35} + \mathbf{405} + \mathbf{2695}$ . The **1** dimensional representation (rep) corresponds to an SU(6) symmetric operator, while the **35**, **405**, and **2695** dimensional reps characterize respectively, first, second, and third order SU(6) symmetry breaking. Therefore, a general SU(6) symmetry breaking operator for ground state baryons has the form

$$\tilde{\Omega} = \tilde{\Omega}^{\mathbf{35}} + \tilde{\Omega}^{\mathbf{405}} + \tilde{\Omega}^{\mathbf{2695}}. \quad (2)$$

The second step is to decompose each SU(6) tensor  $\tilde{\Omega}^R$  in Eq.(2) into SU(3)<sub>F</sub> × SU(2)<sub>J</sub> subtensors  $\tilde{\Omega}_{(F,2J+1)}^R$ , where  $F$  and  $2J+1$  are the dimensionalities of the flavor and spin reps. One finds [4, 9] that a flavor singlet ( $F = 1$ ) axial vector ( $J = 1$ ) operator  $\tilde{\Omega}_{(1,3)}^R$  needed to describe baryon spin, is contained *only* in the  $R = \mathbf{35}$  and  $R = \mathbf{2695}$  dimensional reps of SU(6).

The third step is to construct quark operators transforming as the SU(6) tensor  $\tilde{\Omega}_{(1,3)}^R$ . In terms of quarks, the SU(6) tensors on the right-hand side of Eq.(2) are represented respectively by one-, two-, and three-quark operators [10]. We find the following uniquely determined one-quark  $\mathbf{A}_{[1]}$  and three-quark  $\mathbf{A}_{[3]}$  flavor singlet axial currents [4]

$$\begin{aligned} \tilde{\Omega}_{(1,3)}^{\mathbf{35}} &= \mathbf{A}_{[1]} = A \sum_{i=1}^3 \sigma_i, \\ \tilde{\Omega}_{(1,3)}^{\mathbf{2695}} &= \mathbf{A}_{[3]} = C \sum_{i \neq j \neq k}^3 \sigma_i \cdot \sigma_j \sigma_k, \end{aligned} \quad (3)$$

where  $\sigma_i$  is the Pauli spin matrix of quark  $i$ . The constants  $A$  and  $C$  are to be determined from experiment. The most general flavor singlet axial current compatible

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with broken SU(6) symmetry is then

$$\mathbf{A} = \mathbf{A}_{[1]} + \mathbf{A}_{[3]} = A \sum_{i=1}^3 \boldsymbol{\sigma}_i + C \sum_{i \neq j \neq k}^3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\sigma}_k. \quad (4)$$

The additive quark model corresponds to  $C = 0$  and  $A = 1$ . The three-quark operators are an effective description of quark-antiquark and gluon degrees of freedom.

### III. QUARK SPIN CONTRIBUTION TO BARYON SPIN

By sandwiching the flavor singlet axial current  $\mathbf{A}$  of Eq.(4) between standard SU(6) baryon wave functions [11] we obtain for the quark spin contribution to the spin of octet and decuplet baryons [4]

$$\begin{aligned} \Sigma_1 &= \langle B_8 \uparrow | \mathbf{A}_z | B_8 \uparrow \rangle = A - 10C, \\ \Sigma_3 &= \langle B_{10} \uparrow | \mathbf{A}_z | B_{10} \uparrow \rangle = 3A + 6C, \end{aligned} \quad (5)$$

where  $B_8$  ( $B_{10}$ ) stands for any member of the baryon flavor octet (decuplet). Here,  $\Sigma_1$  ( $\Sigma_3$ ) is twice the quark spin contribution to octet (decuplet) baryon spin. Our theory predicts the same quark contribution to baryon spin for all members of a given flavor multiplet, because the operator in Eq.(4) is by construction a flavor singlet that does not break SU(3) flavor symmetry. On the other hand, SU(6) spin-flavor symmetry is broken as reflected by the different expressions for flavor octet and decuplet baryons.

To proceed, we construct from the operators in Eq.(4) one-body  $\mathbf{A}_{[1]z}^q$  and three-body  $\mathbf{A}_{[3]z}^q$  operators of flavor  $q$  acting only on  $u$  quarks and  $d$  quarks [4]

$$\begin{aligned} \mathbf{A}_z^u &= A \sum_{i=1}^3 \boldsymbol{\sigma}_{iz}^u + 2C \sum_{i \neq j \neq k}^3 \boldsymbol{\sigma}_i^u \cdot \boldsymbol{\sigma}_j^u \boldsymbol{\sigma}_{kz}^u, \\ \mathbf{A}_z^d &= A \sum_{i=1}^3 \boldsymbol{\sigma}_{iz}^d + C \sum_{i \neq j \neq k}^3 \boldsymbol{\sigma}_i^u \cdot \boldsymbol{\sigma}_j^u \boldsymbol{\sigma}_{kz}^d. \end{aligned} \quad (6)$$

For the  $u$  and  $d$  quark contributions to the spin of the proton we obtain

$$\begin{aligned} \Delta u &= \langle p \uparrow | \mathbf{A}_{[1]z}^u + \mathbf{A}_{[3]z}^u | p \uparrow \rangle = \frac{4}{3}A - \frac{28}{3}C, \\ \Delta d &= \langle p \uparrow | \mathbf{A}_{[1]z}^d + \mathbf{A}_{[3]z}^d | p \uparrow \rangle = -\frac{1}{3}A - \frac{2}{3}C. \end{aligned} \quad (7)$$

These theoretical results are to be compared with the combined deep inelastic scattering and hyperon  $\beta$ -decay experimental data, from which the following quark spin contributions to the proton spin were extracted [3]  $\Delta u = 0.84 \pm 0.03$ ,  $\Delta d = -0.43 \pm 0.03$ ,  $\Delta s = -0.08 \pm 0.03$ . The sum of these spin fractions  $\Sigma_{1_{exp}} = \Delta u + \Delta d + \Delta s = 0.33(08)$  is considerably smaller than expected from the additive quark model, which gives  $\Sigma_1 = 1$ .

Solving Eq.(7) for  $A$  and  $C$  fixes the constants  $A$  and  $C$  as

$$A = \frac{1}{6} \Delta u - \frac{7}{3} \Delta d, \quad C = -\frac{1}{12} \Delta u - \frac{1}{3} \Delta d. \quad (8)$$

Inserting the experimental results for  $\Delta u$  and  $\Delta d$  we obtain  $A = 1.143(70)$  and  $C = 0.073(10)$  and from Eq.(5)

$$\begin{aligned} \Sigma_1 &= A - 10C = 1.14 - 0.73 = 0.41(12), \\ \Sigma_3 &= 3A + 6C = 3.42 + 0.45 = 3.87(22) \end{aligned} \quad (9)$$

compared to the experimental result  $\Sigma_{1_{exp}} = 0.33(08)$ . For octet baryons, the three-quark term is of the same importance as the one-quark term because of the factor 10 multiplying  $C$ . It is interesting that for decuplet baryons, quark spins add up to 1.3 times the additive quark model value  $\Sigma_3 = 3$ .

### IV. QUARK ORBITAL ANGULAR MOMENTUM CONTRIBUTION TO BARYON SPIN

In this section we apply the spin-flavor operator analysis of Sect. III) to quark orbital angular momentum  $L_z$  using the general operator of Eq.(4) for  $L_z$  with new constants  $A'$  and  $C'$

$$\begin{aligned} L_z(8) &= \langle B_8 \uparrow | L_z | B_8 \uparrow \rangle = \frac{1}{2} (A' - 10C'), \\ L_z(10) &= \langle B_{10} \uparrow | L_z | B_{10} \uparrow \rangle = \frac{1}{2} (3A' + 6C'). \end{aligned} \quad (10)$$

Assuming that the gluon total angular momentum  $S_g + L_g \approx 0$  is small [3] we obtain from Eq.(1)

$$\begin{aligned} L_z(8) &= \frac{1}{2} - \frac{1}{2} \Sigma_1 = 0.30, \\ L_z(10) &= \frac{3}{2} - \frac{1}{2} \Sigma_3 = -0.44. \end{aligned} \quad (11)$$

Eq.(10) and Eq.(11) yield for the parameters  $A' = 1 - A = -0.143$  and  $C' = -C = -0.073$ . Next, we calculate the orbital angular momentum carried by  $u$  and  $d$  quarks in the proton in analogy to Eq.(7)

$$\begin{aligned} L_z^u(p) &= \frac{1}{2} \left( \frac{4}{3} A' - \frac{28}{3} C' \right) = 0.25, \\ L_z^d(p) &= \frac{1}{2} \left( -\frac{1}{3} A' - \frac{2}{3} C' \right) = 0.05. \end{aligned} \quad (12)$$

For the total angular momentum carried by quarks we get  $J^u(p) = \frac{1}{2} \Delta u + L_z^u(p) = 0.42 + 0.25 = 0.67$  and  $J^d(p) = \frac{1}{2} \Delta d + L_z^d(p) = -0.22 + 0.05 = -0.17$ . Our results for  $J^u(p)$  and  $J^d(p)$  are consistent with those of Thomas [12] who finds  $J^u(p) = 0.67$  and  $J^d(p) = -0.17$  at the low energy (model) scale. Applying the  $u$  and  $d$  quark operators in Eq.(6) to the  $\Delta^+$  state we obtain

$$\begin{aligned} L_z^u(\Delta^+) &= \frac{1}{2} (2A' + 4C') = -0.29, \\ L_z^d(\Delta^+) &= \frac{1}{2} (A' + 2C') = -0.15. \end{aligned} \quad (13)$$

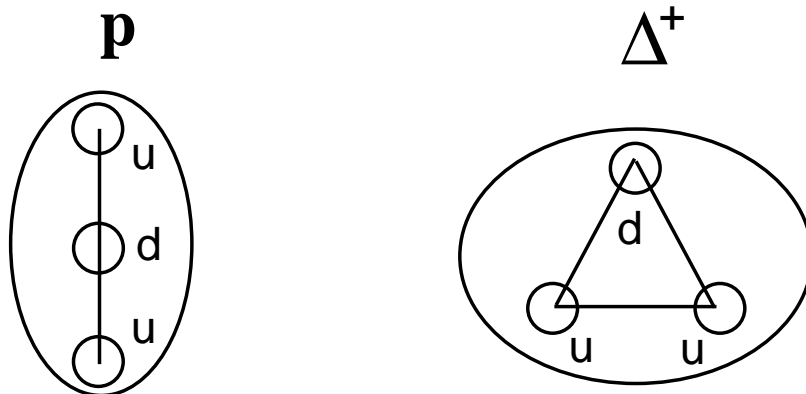


FIG. 1: Qualitative picture of the  $u$  and  $d$  quark distributions in the proton (left) and  $\Delta^+$  (right). In the proton, most of the quark orbital angular momentum is carried by the  $u$  quarks and relatively little by the  $d$  quarks. This is consistent with a linear (prolate or cigar-shaped) quark distribution with the  $u$  quarks at the periphery and the  $d$  quark near the origin. In contrast, in the  $\Delta^+$ , the  $u$  quark orbital angular momentum is just twice that of the  $d$  quark. This is consistent with a planar (oblate or pancake-shaped) quark distribution, in which each quark has the same distance from the origin.

We suggest an interpretation of Eq.(12) and Eq.(13) in terms of the geometric shapes of these baryons as depicted in Fig. 1. Previously, by studying the electromagnetic  $p \rightarrow \Delta^+$  transition in various baryon structure models, we have found that the proton has a positive intrinsic quadrupole moment  $Q_0(p)$  corresponding to a

prolate intrinsic charge distribution whereas the  $\Delta^+$  has a negative intrinsic quadrupole moment of similar magnitude  $Q_0(\Delta^+) \approx -Q_0(p)$  corresponding to an oblate charge distribution [7]. This appears to be consistent with our present findings for the quark orbital angular momenta  $L_z^u$  and  $L_z^d$  in both systems.

## V. SUMMARY

In summary, using a broken spin-flavor symmetry based parametrization of QCD, we have presented a straightforward calculation of the quark spin and orbital angular momentum contributions to the total baryon spin. For flavor octet baryons, we have shown that three-quark operators reduce the standard quark model prediction based on one-quark operators from  $\Sigma_1 = 1$  to  $\Sigma_1 = 0.41(12)$  in agreement with the experimental result. On the other hand, in the case of flavor decuplet baryons, three-quark operators enhance the contribution

of one quark operators from  $\Sigma_3 = 3$  to  $\Sigma_3 = 3.87(22)$ .

Assuming that the gluon contribution to baryon spin is small, we have suggested a qualitative interpretation of the positive and large  $u$  quark and small  $d$  quark orbital angular momenta in the proton in terms of a prolate quark distribution corresponding to a positive intrinsic quadrupole moment. In the case of the  $\Delta^+$ ,  $u$  and  $d$  quarks have negative orbital angular momenta of the same magnitude corresponding to an oblate quark distribution giving rise to a negative intrinsic quadrupole moment.

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